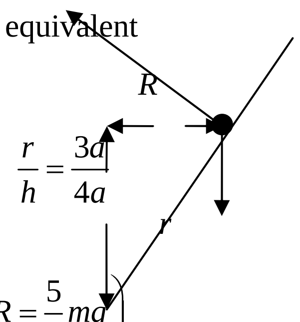
June 2006
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.	Use of $(\pi)\int y^2\,dx \times \bar{x} = (\pi)\int xy^2\,dx$	M1
	$\int x\,dx \times \bar{x} = \int x^2\,dx$	
	$\left[\frac{1}{2}x^2\right]_{\dots}^{\dots} \times \bar{x} = \left[\frac{1}{3}x^3\right]_{\dots}^{\dots}$	A1 = A1
	Using limits 0 and 4 $\quad \frac{16}{2} \times \bar{x} = \frac{64}{3}$	M1
	$\bar{x} = \frac{8}{3}$	A1 **(5)**
		[5]
2.	(a) Small Hemisphere / Bowl / Large Hemisphere	
	Mass ratios $\quad \frac{2}{3}\pi\left(\frac{a}{2}\right)^3 \quad \frac{2}{3}\pi\frac{7a^3}{8} \quad \frac{2}{3}\pi a^3$	B1
	Anything in the ratio 1 : 7 : 8	
	$\bar{x} \quad \frac{3}{16}a \quad \bar{x} \quad \frac{3}{8}a$	B1
	$1 \times \frac{3}{16}a + 7 \times \bar{x} = 8 \times \frac{3}{8}a$	M1 A1
	Leading to $\quad \bar{x} = \frac{45}{112}a$ * $\quad$ cso	A1 **(5)**
	(b) Bowl / Liquid / Bowl and Liquid	
	Mass Ratios $\quad M \quad kM \quad (k+1)M$	B1
	$\bar{x} \quad \frac{45}{112}a \quad \frac{3}{16}a \quad \frac{17}{48}a$	B1
	$M \times \frac{45}{112}a + kM \times \frac{3}{16}a = (k+1)M \times \frac{17}{48}a$	M1 A1
	Leading to $\quad k = \frac{2}{7}$	A1 **(5)**
		[10]

Question Number	Scheme	Marks
3.	(a) $a = 0.1$	B1
	$\frac{2\pi}{\omega} = \frac{1}{5} \Rightarrow \omega = 10\pi$	M1 A1
	$F_{\max} = ma\omega^2$	M1
	$= 0.2 \times 0.1 \times (10\pi)^2$	M1
	$\approx 19.7 \quad (\text{N})$	A1
	cao	**(6)**
	(b) $a' = 0.2, \quad \omega' = 10\pi$	B1ft, B1ft
	$v^2 = \omega^2(a^2 - x^2) = 100\pi^2(0.2^2 - 0.1^2) \quad (= 3\pi^2 \approx 29.6\ ...)$	M1 A1
	$v \approx 5.44 \quad (\text{m s}^{-1})$	A1
	cao	**(5)**
	If answers are given to more than 3 significant figures a maximum of one A mark is lost in the question.	**[11]**
4.	$\tan\alpha = \frac{3}{4}$	B1
	or equivalent	
		
	$\tan\alpha = \frac{r}{h}$ or $\frac{r}{h} = \frac{3a}{4a}$	B1
	$R(\uparrow) \quad R\sin\alpha = mg \quad \left(R = \frac{5}{3}mg\right)$	M1 A1
	$R(\leftarrow) \quad R\cos\alpha = mr\omega^2$	M1 A1
	$= mr \times \frac{8g}{9a} \quad \left(R = \frac{10mrg}{9a}\right)$	A1
	Eliminating R: $\tan\alpha = \frac{9a}{8r} \quad \left(\frac{5}{3}mg = \frac{10mrg}{9a}\right)$	M1 A1
	$\left(\frac{3}{4} = \frac{9a}{8r} \Rightarrow r = \frac{3}{2}a\right)$	
	$h = \frac{r}{\tan\alpha} = \frac{3a}{2} \times \frac{4}{3} = 2a$	M1 A1
		(11)

		[11]

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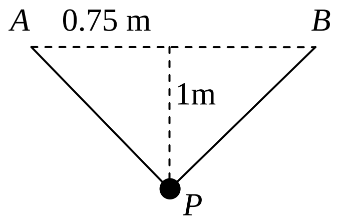
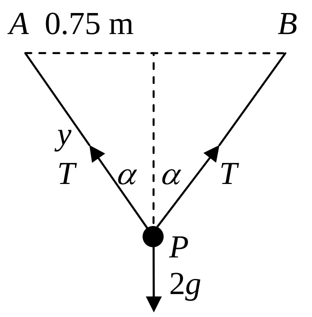
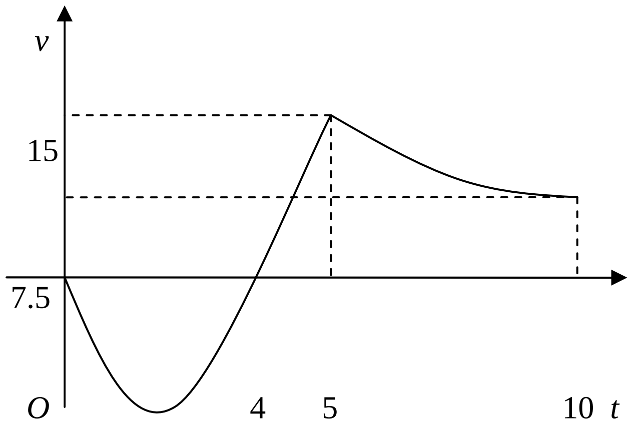
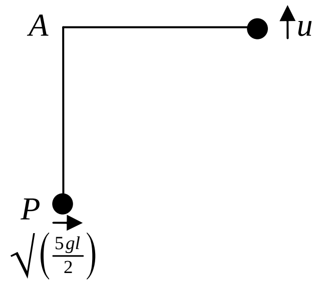
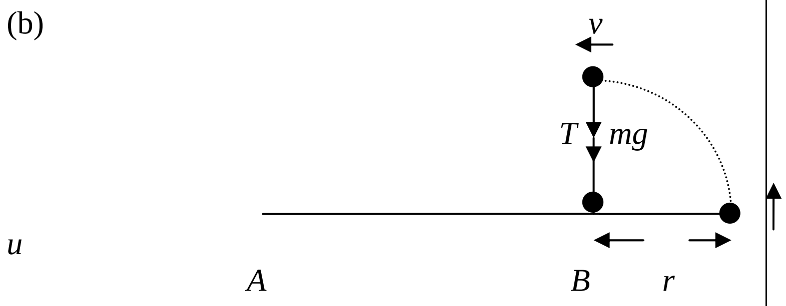
5.	(a) 	
	$AP = \sqrt{(0.75^2 + 1^2)} = 1.25$	M1 A1
	Conservation of energy $\frac{1}{2} \times 2 \times v^2 + 2 \times \frac{49 \times 0.5^2}{2 \times 0.75} = 2g \times 1$ −1 for each incorrect term	M1 A2 (1, 0)
	Leading to $v \approx 1.8$ (m s^{-1}) accept 1.81	A1 **(6)**
	(b) 	
	$R(\uparrow)$ $2T\cos\alpha = 2g$	M1 A1
	$y = \frac{0.75}{\sin\alpha}$	
	Hooke's Law $T = \frac{49}{0.75}\left(\frac{0.75}{\sin\alpha} - 0.75\right)$	M1 A1
	$= 49\left(\frac{1}{\sin\alpha} - 1\right)$	
	$\frac{9.8}{\cos\alpha} = 49\left(\frac{1}{\sin\alpha} - 1\right)$	M1
	Eliminating T	
	$\tan\alpha = 5(1 - \sin\alpha)$	
	$5 = \tan\alpha + 5\sin\alpha$ ∗ cso	A1 **(6)**
		[12]

Question Number	Scheme	Marks
6.	(a)	
		
	Parabola	B1
	Hyperbola	B1
	Points	B1 **(3)**
	(b) Identifying the minimum point of the parabola and 5 as the end points.	M1
	$2 < t < 5$	A1 **(2)**
	(c) Splitting the integral into two part, with limits 0 and 4, and 4 and 5, and evaluating both integrals.	M1
	$\int_0^4 3t(t-4)\,dt = \left[t^3 - 6t^2\right]_0^4 = -32$ and $\int_4^5 3t(t-4)\,dt = \left[t^3 - 6t^2\right]_4^5 = 7$ Both	A1
	Total distance $= 39 \quad (\text{m})$ ∗ cso	A1 **(3)**
	(d) $\int_5^{t_1} \frac{75}{t}\,dt = 32 - 7$	M1 A1
	$75[\ln t]_5^{t_1} = 25$	A1
	$\ln \frac{t_1}{5} = \frac{1}{3} \quad \Rightarrow \quad t_1 = 5e^{\frac{1}{3}}$	M1
	≈ 6.98 cao	A1 **(5)**
		[13]

Question Number	Scheme	Marks
7.	(a)	
		
	Conservation of Energy	
	$\frac{1}{2}m\left(\frac{5gl}{2}-u^2\right)=mgl$	M1 A1= A1
	Leading to $u=\sqrt{\left(\frac{gl}{2}\right)}$	A1 **(4)**
	(b)	
		
	Conservation of Energy	
	$\frac{1}{2}m\left(u^2-v^2\right)=mgr$	M1 A1
	$v^2=u^2-2gr$	
	$R(\downarrow)\quad T+mg=\frac{mv^2}{r}$	M1 A1
	$T=\frac{m}{r}\left(u^2-2gr\right)-mg$	M1
	$=\frac{mu^2}{r}-3mg$	A1
	$=\frac{mgl}{2r}-3mg$	M1
	$T\geq 0\Rightarrow\frac{mgl}{2r}\geq 3mg$	M1
	$\Rightarrow\frac{1}{6}\geq r$	
	$AB_{\text{MIN}}=\frac{5l}{6}$	A1 **(9)**
		[13]